

Metal speciation, complexation: solution

1) Complexometric titration

a) Calculate the total hardness of the sample using the following equations:

$$H \text{ (mol/L)} = \frac{V_L C_L}{V_0} = \frac{2.6 * 0.05}{50} = 0.0025 \text{ M} = 2.5 \text{ mM}$$

Where V_L = volume of EDTA solution used (mL)

C_L = concentration of EDTA solution (M)

V_0 = volume of sample (mL)

The water is classified as intermediate.

b) Goal: formula for $[M]$ as a function of known concentrations and constants.

We define: $H_2EDTA = H_2Y$ and $[Y]_{tot} = [Y]_{added}$

$$K_1 = \frac{[HY^-][H^+]}{[H_2Y]} = 10^{-6.2} \quad \text{eq. 1}$$

$$K_2 = \frac{[Y^{2-}][H^+]}{[HY^-]} = 10^{-10.3} \quad \text{eq. 2}$$

$$[Y]_{tot} = [MY] + [Y] + [HY] + [H_2Y] \quad \text{eq. 3}$$

Express $[HY]$ and $[H_2Y]$ using eqs. 1 and 2 to get:

$$[Y]_{tot} = [MY] + [Y] \left(1 + \frac{[H^+]}{K_2} + \frac{[H^+]^2}{K_1 K_2} \right)$$

Introduce $\left(1 + \frac{[H^+]}{K_2} + \frac{[H^+]^2}{K_1 K_2} \right) = a_H$ and solve for $[Y]$:

$$[Y] = \frac{[Y]_{tot} - [MY]}{a_H} \quad \text{eq. 4}$$

4

$$[M]_{tot} = [MY] + [M]$$

$$[M] = [M]_{tot} - [MY] \quad \text{eq. 5}$$

We do not have the concentration of $[MY]$, we need to express it as a function of the total concentrations and available constants:

$$K_{MY} = \frac{[MY]}{[Y][M]} \quad \text{eq. 6}$$

Substitute $[Y]$ (eq. 4) and $[M]$ (eq. 5) in eq. 6 to get

$$K_{MY} = \frac{[MY] a_H}{([M]_{tot} - [MY])([Y]_{tot} - [MY])}$$

Re-structure to the form of a quadratic equation: $ax^2 + bx + c = 0$ with $x = [MY]$

$$[MY]^2 + [MY]\left([Y]_{\text{tot}} - [M]_{\text{tot}} - \frac{a_H}{K_{MY}}\right) + [Y]_{\text{tot}}[M]_{\text{tot}} = 0$$

Solve equation for $[MY]$: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$[MY] = \frac{[Y]_{\text{tot}} + [M]_{\text{tot}} + \frac{a_H}{K_{MY}} - \sqrt{\left([Y]_{\text{tot}} + [M]_{\text{tot}} + \frac{a_H}{K_{MY}}\right)^2 - 4[Y]_{\text{tot}}[M]_{\text{tot}}}}{2} \quad \text{eq. 7}$$

c) Calculate $[MY]$ using eq. 7 above. pH 2 means $[H^+] = 10^{-2} \text{ M}$

For a successful complexometric titration, a high pH and high K are required (higher concentrations of $[MY]$).

pH	$[MY]$	$\log K_{MY}$
2	0	10
5	$2.96 \cdot 10^{-9} \text{ M}$	10
7	$6.21 \cdot 10^{-7} \text{ M}$	10
10	$9.83 \cdot 10^{-7} \text{ M}$	10
5	$4.32 \cdot 10^{-11} \text{ M}$	5
10	$6.21 \cdot 10^{-7} \text{ M}$	10

2) Lead speciation

a) Speciation

$$[Pb]_{\text{tot}} = [Pb^{2+}] + [PbOH^+] + [PbCO_3] = [Pb^{2+}](1 + 10^{6.3} [OH^-] + 10^{6.2} [CO_3^{2-}])$$

$$[CO_3^{2-}]_{\text{tot}} = [CO_3^{2-}] + [HCO_3^-]$$

Thus, $[HCO_3^-] = [CO_3^{2-}]_{\text{tot}} - [CO_3^{2-}]$

$$K_a = 10^{-10.3} = \frac{[CO_3^{2-}][H^+]}{[HCO_3^-]} = \frac{[CO_3^{2-}][H^+]}{[CO_3^{2-}]_{\text{tot}} - [CO_3^{2-}]}$$

$$K_a[CO_3^{2-}]_{\text{tot}} - K_a[CO_3^{2-}] = [CO_3^{2-}][H^+]$$

$$[CO_3^{2-}] = \frac{K_a[CO_3^{2-}]_{\text{tot}}}{K_a + [H^+]} = \frac{10^{-10.3}[CO_3^{2-}]_{\text{tot}}}{10^{-10.3} + 10^{-7.8}} = 4.1 \cdot 10^{-6} \text{ M}$$

Hence, $[Pb]_{\text{tot}} = [Pb^{2+}](1 + 1.26 + 6.50) = [Pb^{2+}] \cdot 8.76$

$$[Pb^{2+}] = 1.14 \cdot 10^{-10} \text{ M}$$

$$[PbOH^+] = 1.44 \cdot 10^{-10} \text{ M}$$

$$[\text{Pb}(\text{CO}_3)_{\text{aq}}] = 7.41 * 10^{-10} \text{ M}$$

b) Addition of EDTA

$$[\text{Y}]_{\text{tot}} = [\text{Y}] + [\text{HY}] + [\text{PbY}] + [\text{CaY}]$$

As PbY is a strong complex: $[\text{PbY}] \approx [\text{Pb}]_{\text{tot}} = 10^{-9} \text{ M}$

$$[\text{Y}]_{\text{tot}} - [\text{Pb}]_{\text{tot}} = [\text{Y}] + [\text{HY}] + [\text{CaY}]$$

$$\text{Acid-base equilibrium: } \frac{[\text{Y}][\text{H}^+]}{[\text{HY}]} = 10^{-10.2}$$

$$\text{Complexation of Ca}^{2+}: \frac{[\text{CaY}]}{[\text{Ca}^{2+}][\text{Y}]} = 10^{10.7}$$

$$[\text{Ca}^{2+}]_{\text{tot}} = [\text{Ca}^{2+}] + [\text{CaY}]$$

Since $[\text{CaY}] \ll [\text{Ca}^{2+}] \sim [\text{Ca}^{2+}]_{\text{tot}}$

$$[\text{Y}]_{\text{tot}} - [\text{Pb}]_{\text{tot}} = [\text{Y}]\left(1 + \frac{[\text{H}^+]}{10^{-10.2}}\right) + [\text{CaY}]$$

$$[\text{Y}]_{\text{tot}} - [\text{Pb}]_{\text{tot}} = [\text{Y}]\left(1 + \frac{[\text{H}^+]}{10^{-10.2}}\right) + 10^{10.7}[\text{Ca}^{2+}]_{\text{tot}}[\text{Y}]$$

$$[\text{Y}]_{\text{tot}} - [\text{Pb}]_{\text{tot}} = [\text{Y}]\left(1 + \frac{[\text{H}^+]}{10^{-10.2}} + 10^{10.7}[\text{Ca}^{2+}]_{\text{tot}}\right)$$

Solve for [Y] to get $[\text{Y}] = 1.98 * 10^{-15} \text{ M}$

$$[\text{Pb}]_{\text{tot}} = [\text{Pb}^{2+}] (1 + 10^{6.3}[\text{OH}^-] + 10^{6.2}[\text{CO}_3^{2-}] + 10^{19.8}[\text{Y}])$$

Solve for $[\text{Pb}^{2+}]$ to get $[\text{Pb}^{2+}] = 8 * 10^{-15} \text{ M}$

3) Hydrolysis of Cu^{2+} and complex formation with CO_3^{2-}

$$C_{\text{T}} = [\text{H}_2\text{CO}_3] + [\text{HCO}_3^-] + [\text{CO}_3^{2-}] + [\text{CuCO}_3] + 2 [\text{Cu}(\text{CO}_3)_2^{2-}]$$

The last two terms can be neglected. Thus, we first calculate the CO_3^{2-} concentration

for pH = 8:

$$C_{\text{T}} = [\text{CO}_3^{2-}] \left(\frac{[\text{H}^+]^2}{K_{a1}K_{a2}} + \frac{[\text{H}^+]}{K_1} + 1 \right)$$

$$[\text{CO}_3^{2-}] = 9.8 * 10^{-6} \text{ M}$$

Now we can compute the individual Cu(II) species:

$$[\text{Cu}]_{\text{T}} = [\text{Cu}^{2+}] + [\text{CuOH}^+] + [\text{Cu}(\text{OH})_2] + [\text{Cu}(\text{OH})_3^-] + [\text{Cu}(\text{OH})_4^{2-}] + [\text{CuCO}_3] + [\text{Cu}(\text{CO}_3)_2^{2-}]$$

$$[\text{Cu}]_{\text{T}} = [\text{Cu}^{2+}] (1 + \beta_1[\text{H}^+]^{-1} + \beta_2[\text{H}^+]^{-2} + \beta_3[\text{H}^+]^{-3} + \beta_4[\text{H}^+]^{-4} + \beta_{1\text{CO}_3}[\text{CO}_3^{2-}] + \beta_{2\text{CO}_3}[\text{CO}_3^{2-}]^2)$$

For pH = 8.0, we obtain the following distribution

Species	Concentration (M)	% Cu _T
Cu ²⁺	8.2 * 10 ⁻¹⁰	1.6
CuOH ⁺	8.2 * 10 ⁻¹⁰	1.6
Cu(OH) ₂	5.2 * 10 ⁻¹⁰	1.0
Cu(OH) ₃ ⁻	1.3 * 10 ⁻¹²	3 * 10 ⁻³
Cu(OH) ₄ ²⁻	1 * 10 ⁻¹⁷	8 * 10 ⁻⁸
CuCO ₃	4.7 * 10 ⁻⁸	94.0
Cu(CO ₃) ₂ ²⁻	7.8 * 10 ⁻¹⁰	1.6